MANCHESTER

LIFE TABLE.

1881-90.

JOHN TATHAM

Sir Andrew Clark. Bart.
With the Writers respectful Complim[ents]

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BY

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MANCHESTER:

H. BLACKLOCK & CO., PRINTERS, ALBERT SQUARE.

1892.

PREFACE.

Long before the census-taking of 1891, I resolved that my first project on receiving the local details of that census should be the construction of a life table for Manchester. In the year 1883, Mr. Noel A. Humphreys, now Secretary of the Census Office, read before the Statistical Society a very able paper, "On the recent decline in the English death-rate, and its effect upon the duration of life." In that paper important conclusions were drawn from a life table, prepared for the purpose, and based on the population and death figures in England and Wales during the years 1876–80. Attentive study of that essay has established in my mind the conviction, which under Mr. Humphreys' guidance had been steadily gaining strength of late years, that a local life table must henceforth be regarded as indispensable to every Medical Officer of Health. It is to him—if one may use a homely simile—what the "two-foot rule" is to the mechanic. In a word, the life table is the one and only means by which the vague expressions "more" or "less" of the sanitarian can be reduced to an exact comparative standard.

And whilst speaking of Mr. Humphreys in connection with life table work, let me say how great is my personal indebtedness to him for the many acts of kindness and for the valuable statistical aid which he has rendered to me during more than seventeen years of official life. Himself a pupil and a trusted friend of the late Dr. Farr's, he has inherited much of the ability and enthusiasm of that great master in regard to the popularisation of the science of vital statistics; whilst in his constant readiness to impart statistical knowledge to Health Officers and others who so greatly need it, he stands second not even to Dr. Farr himself.

Unfortunately the complete results of the census enumeration are not ordinarily published for the several registration areas of England until between two and three years after the enumeration has taken place: consequently the first essential for the construction of a life table—namely, an accurate return of the sexes and ages of the population—was, under existing circumstances, unattainable before the publication of the detailed census report. In this dilemma, I applied for help to the Registrar General; and I beg leave to acknowledge, in this place, my obligation to Sir Brydges Henniker for the readiness with which he complied with my request for the necessary particulars, in anticipation of their presentation to Parliament. Within a short period of my application, I was supplied with a detailed statement of the numbers living in Manchester at the time of the census, and also of the deaths registered in the previous decennium, distinguishing sex and age in each case.

Having thus obtained the crude materials for the construction of a Manchester Life Table, the next step was to decide how these could be so

dealt with that, on the one hand they should be made to yield the greatest amount of useful information, and that, on the other hand, the results obtained should be unimpeachable, or at any rate should be free from avoidable error. Fully conscious as I was, *in limine*, of the serious amount of arithmetical labour involved in manipulating the huge mass of figures which go to form the basis of my projected life table, I felt that, consistently with the discharge of other duties, it would be impossible for me to undertake it without the co-operation of an expert in these matters. And, in addition to this, I was convinced that work of so complex and technical a character as that which was then in contemplation, could be undertaken with fair prospect of success only by an experienced mathematician specially skilled in methods of life table construction.

Under these circumstances, I had the good fortune to secure the assistance of Mr. A. C. Waters, of the Statistical Department of the General Register Office; and I desire here to express my grateful appreciation of the ability and zeal he has shown in grappling with the difficulties which are known to be inseparable from work of this kind. Mr. Waters has been good enough to undertake for me all the mathematical work of the tables at the end of this pamphlet: and in this connection I wish specially to direct attention to certain novel mathematical and other devices which will be found interspersed through the letterpress; notably at pages 3-4, in the foot-note to page 16, and at page 25. These devices I believe to be original, and I have to thank their author for permitting me to use them for the first time in connection with my new Manchester Life Table. Mr. Waters has further been good enough to examine the whole of the proof-sheets, and has in other ways rendered me valuable assistance in connection with the work.

JOHN TATHAM.

Public Health Office,
Town Hall, Manchester,
August, 1892.

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I.—General description of a Life Table.

Imagine that in the year 1782, or just 110 years ago, a great ledger containing 100,000 pages had been opened, and had been kept assiduously and without error ever since that date. Imagine further, that on each page of our ledger the name of a child, with the date of its birth, had been inscribed, and that every anniversary of a birthday had been noted on the appropriate page until the inevitable day when the account had to be marked "closed." It is certain that before now nearly every one of the 100,000 lives would have run its course. Now let us turn over the leaves of our ledger and mark on a sheet of paper how many lives failed before completing one year of age, how many after the first birthday but before the second—and so on throughout the series. The figures so obtained form one of the fundamental columns of a life table.

Again, if the ledger showed the *date* of each death as well as the number of birthdays attained, we could state the *actual lifetime* of each person, in years, months, and days. The addition of these 100,000 ages would give the *total lifetime* of the 100,000 persons, and the $\frac{1}{100000}$th part of this total lifetime would be the *average lifetime* of each person. Moreover, if another similar ledger were opened under like circumstances, this *average lifetime* would express the *expectation of life* of each infant included in this new ledger.

Now it is evident that the numbers standing against the successive ages in the table would vary (*a*) according to the vitality possessed by the 100,000 children at birth, and also (*b*) according to the conditions by which they were surrounded in after years. Feeble initial vitality, unhealthy surroundings, or dangerous occupations, would produce large numbers in the column of deaths, and rapidly decreasing numbers in the column of survivors; so that only a few stragglers would be left to attain to the higher ages. On the other hand, under the influence of more favourable conditions at birth and more healthful surroundings in after life, the deaths at the earlier ages would be relatively few, the survivors would decrease more slowly, and larger numbers would remain at the more advanced periods of life. We may imagine a life table of persons whose whole lives were passed in a particular locality or in a particular class of society, or among conditions

incidental to a particular trade or calling. It would, however, be practically impossible to construct, on a sufficiently large basis, a life table precisely answering to either of these conditions; and even if it were possible, it is obvious that such a table, commenced forthwith and carried on unremittingly by the tedious process of recording the deaths and survivors year after year, could be completed only by our distant successors about the end of the twentieth century. And, further, there would be danger of possible error in using the results of the table as a forecast. Suppose, for instance, that the 100,000 births were spread over five years, and that when the survivors were all between 10 and 15 years of age some destructive epidemic carried off a large number of them; in that event it would appear that the rates of dying at these ages were very high, and the table might be used to show that this was a period of life at which the vitality was particularly low. Such an error might be avoided by making records of several other 100,000's, *commenced at different times*, and aggregating the results; or even by making up one 100,000 of (say) ten batches of 10,000 each, commenced at different times. It would be unlikely that *each* batch would reach a similar epidemic year at the same period of life. It now becomes important to inquire whether or not we can form such a table without the help of prolonged records; and, if we can, whether a life table thus readily constructed would equally well answer our purpose. We shall be able to show that it is possible, with the aid of mathematical devices, to construct a table, based on the records of a few years, which shall afford an accurate measure of the life conditions prevailing during those years.

II.—Basis of facts for a Life Table for Manchester.

Such a table as that above described has been constructed, with the object of tracing the conditions affecting human life in Manchester in the ten years 1881-90: of comparing these with previous conditions, and with the conditions at various times in the country at large; and, lastly, of obtaining a means of correcting future death-rates and showing their real significance.

The life table is based on two sets of facts:—first, the populations enumerated at the censuses of 1881 and 1891; and, secondly, the deaths registered in the ten years 1881-90. It is now proposed to show how these two sets of facts have been brought into relation with each other, and made to yield a table of the same form and with the same general meaning as the table we have supposed to have been slowly built up by the patience of generations of record keepers.

It was originally intended to make the life table relate exclusively to the area of the City of Manchester as enlarged by the Act of 1890. But this project had to be abandoned because of the impossibility of obtaining accurate records of deaths in past years in this new area, which, as a whole, corresponds with no district, or group of districts, hitherto recognised by

the Registrar General, but has been arbitrarily carved out of three distinct, though adjacent, Poor Law Unions in the County of Lancaster. It was therefore determined, after careful consideration, to take the three entire Unions or Registration Districts of Manchester, Chorlton, and Prestwich, as representing the City with sufficient approximation. It is true that these districts include a wide area which is outside the limits of the City, and in which the conditions of life are mainly rural; but since the great bulk of the population and of the deaths belong to the City, the rates of mortality (on which alone the table is founded) are practically those of Manchester.

The census returns show that in April, 1881, the three Unions or Registration Districts of Manchester, Chorlton, and Prestwich contained 528,307 persons, and that by April, 1891, the number had increased to 594,502. We are told how many of these were males, and how many were females; how many were returned as living in each year of age up to five years, and how many in each subsequent five-year or ten-year period, up to the most advanced age. The death returns show how many of each sex died in certain groups of ages in 1881, how many in the same groups in 1882, and so on.

It is true that we cannot trace the changes in the population from year to year: for, besides the varying numbers of births and deaths, there has been a constant stream of persons flowing into the district from beyond its boundaries, and another stream flowing outwards; and these changes, of which we have no record, have doubtless varied in amount from time to time—such changes depending on complex social, economical, and other causes. But, on the whole, we shall be near the truth if we assume the ten years' growth from 528,307 to 594,502 to have gone on at a uniform rate.

On the assumption of a uniform rate of increase, the mean population in a period during which the population has changed from P to rP may be shown to be equal to

$$\frac{r\text{P}-\text{P}}{\text{Hyperbolic logarithm of } r.} \quad \text{or} \quad \frac{(r\text{P}-\text{P})\times .43429448}{\text{Common logarithm of } r.}$$

Dr. Farr, in his English Life Table No. 3, pointed out that "the "true mean population (calculated by this formula) which, multiplied by the "number of years in the period is equal to the years of life, is less than the "mean of the population living at the beginning and end, but more than "the population living in the middle of the period." He, however, overlooked a remarkably close approximation to the true mean, which may be obtained by adding *one-third* of the mean of the populations at the beginning and end of the period to *two-thirds* of the population in the middle of the period. Take, for instance, the population of the three districts representing Manchester as enumerated in 1881 and 1891, viz., 528,307 and 594,502. By the formula, the true mean population throughout the period was 560,751·8, which, multiplied by 10 (the number of years) gives 5,607,518 years of life. The mean

of the populations at the beginning and end is 561,404·5, giving 5,614,045 years of life; and the population in the middle of the period was 560,428·1, giving 5,604,281 years of life. Adding $\frac{1}{3}$ of 561,404·5 to $\frac{2}{3}$ of 560,428·1, we get 560,753·5, giving 5,607,535 years of life: results which are within 2 of the true mean population, and within 17 of the true number of years of life. The following figures show the closeness of the approximation in an extreme case. Suppose a population to have increased at a uniform rate from 1,000,000 to 2,000,000, in any period. The true mean, by the formula, is 1,442,695.

Mean of population at beginning and end = 1,500,000, which is 57,305 in excess.
Population in middle of period = 1,414,213, which is 28,402 in defect.
$\frac{1}{3}$ of 1,500,000 + $\frac{2}{3}$ of 1,414,213 = 1,442,809, which is 114 in excess.

In this extreme case the several errors, compared with the true mean, are 4 per cent., 2 per cent., and ·008 per cent.; or one in 25, one in 50, and one in 12,655, respectively.* We can, therefore, readily estimate the mean population on the basis of the known populations at the beginning and end of the period.

Further, we know how each 100,000 (on the average) in 1881 was made up of different sexes and ages, and how this average "age and sex distribution" had changed by 1891. We shall not be far wrong if we assume that this known change also went on at a uniform rate. Thus we can calculate the mean age and sex distribution in the period and apply it to the mean population obtained as above.

No doubt if we could learn the actual facts we should find that in some years our estimates were too great, in some too small. In one case we should have overstated, and in another understated, one or other of the age or sex groups. But, if we add together all our estimates, these errors of excess and defect will to a great extent balance each other. If we have counted 1,000 too many in one year and 1,100 too few in another, the addition of the two years gives us an error of only 100, which is enormously less important in proportion to the total of *two* years' population than was 1,000 or 1,100 in proportion to *one* year's population. It is a fact of the greatest importance in statistics that, if a number of observations or estimates be added together, the *total error* cannot be proportionally greater than the *average error* in the separate observations. It can only be *as great* when all the errors are in the same direction, and since, in almost all cases, errors occur in opposite directions, the errors tend to balance one another, often making the total error, not only proportionally but absolutely less than either of the individual errors.

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* If P be the population at the beginning, and r P at the end, and if $i = 1 - r$,

Mean of populations at beginning and end $= P \times (1 + \frac{1}{2}i)$

Population in middle of period $= P \times (1 + \frac{1}{2}i - \frac{1}{8}i^2 + \frac{1}{16}i^3 - \frac{5}{128}i^4 + \text{\&c.})$

True mean population $= P \times (1 + \frac{1}{2}i - \frac{1}{12}i^2 + \frac{1}{24}i^3 - \frac{19}{720}i^4 + \text{\&c.})$

Mean population by approximate method $= P \times (1 + \frac{1}{2}i - \frac{1}{12}i^2 + \frac{1}{24}i^3 - \frac{5}{192}i^4 + \text{\&c.})$;

which agrees with the *true* mean as far as the term involving i^3, and differs in the next term by $\frac{1}{2880}i^4$ only.

As we are dealing with the deaths in the ten years from January, 1881, to December, 1890, our mean population must obviously be calculated for the same period, and not for the ten years from April, 1881, to April, 1891. This has been done on the principles explained above, and the results, together with the numbers of deaths, are shown in the following table :—

TABLE A.

Sex and Age	Enumerated Population		Estimated Mean Population, 1st Jan., 1881, to 31st Dec., 1890	Deaths in the 10 Years 1881–90
	1881	1891		
MALES.				
Under 5 years	35,884	35,588	35,835·5	30,089
5—	30,396	32,562	31,434·3	2,396
10—	26,350	32,153	29,003·4	1,075
15—	24,328	29,591	26,735·1	1,456
20—	24,104	27,024	25,464·3	1,771
25—	42,402	46,772	44,452·4	4,901
35—	30,966	36,239	33,393·4	6,528
45—	20,087	24,361	22,042·6	6,865
55—	11,801	13,147	12,429·4	6,762
65—	4,844	5,832	5,296·4	5,437
75—	1,127	1.249	1,184·2	2,158
85—	63	103	81·0	253
95 and up	1	3	1·7	1
All ages	252,353	284,624	267,353·7	69,692
FEMALES.				
Under 5 years	36,562	36,297	36,527·8	25,859
5—	30,986	33,233	32,061·6	2,373
10—	26,865	32,687	29,525·0	1,089
15—	26,753	32,253	29,268·9	1,437
20—	27,904	31,631	29,633·5	1,810
25—	46,015	51,534	48,593·2	4,615
35—	33,896	38,378	35,977·0	5,557
45—	23,648	27,313	25,340·1	6,159
55—	14.737	16,226	15,436·2	7,042
65—	6,584	8,123	7,286·8	6,406
75—	1,822	1,992	1,902·1	3,039
85—	168	209	186·7	475
95 and up	14	2	8·4	29
All ages	275,954	309,878	291,747·3	65,890
Total of both sexes	528,307	594,502	559,101·0	135,582

By estimating the mean population and dividing it out into sexes and ages, we have avoided a difficulty. In 1881 the census showed 42,402 males

between 25 and 35 years of age. In 1891 the number returned was 46,772. But all the survivors of the 42,402 who were between 25 and 35 years of age in 1881 must obviously have been more than 35 in 1891; and even if we could have traced these 42,402 men through the ten-year period, showing the survivors in 1882 at 26–36 years of age, in 1883 at 27–37, and so on, it would still have been of no use for our purpose, for the records of deaths relate to men aged 25–35 *in each year*. The method we have adopted enables us to compare the *average number of men living* at age 25–35 throughout the whole ten years, with the *average annual deaths* of men in that group of ages, and so on for all other groups. It remains to be shown how these groups of figures can be made to give an approximately correct history of 100,000 infants from birth to old age, and also in what respects this history resembles, and in what it differs from, such a record as that which we have imagined.

It may be taken as matter of common observation that there is no *sudden* change in the rate of mortality at any particular age. The rate is known to be high among newly-born children, and to decrease rapidly for the first few months, then more slowly—but still progressively and not by jerks—until it reaches its minimum: the rate then begins to increase, slowly at first and afterwards more rapidly, until in extreme old age it is even greater than at birth. There is, however, no need to consider all this; we need only claim to avail ourselves of the known fact that the liability to death does not suddenly change at any special period of life.

It will be noticed that the ages in Table A are made up into groups of five and ten years, and that the ten-year groups commence at 25, 35, &c., not at 20, 30, &c. The reason for this method of grouping is that the tendency to state ages "in round numbers" makes returns of *separate ages* utterly useless; and that the same tendency makes the returns in groups 20–30, 30–40, &c., untrustworthy also. This is clear if we consider that large numbers of persons aged 28, 29, 31, or 32 are returned roughly as "30," and would therefore be grouped as 30–40, the errors being all in one direction, since those who had understated their ages would remain in their proper age group, while those who had overstated them would get into a wrong group. The groups 25–35, &c., may, however, be taken as approximately correct. To this rule there are, indeed, one or two exceptions, which chiefly affect the returns of females. Thus, there is a well-known tendency amongst girls a little under 15 years old to overstate their ages, probably with a view to improve their chances of getting employment; and persons in domestic service or employed in business establishments are returned by their employers according to the statements already made. Then, again, there is a tendency among women approaching middle age to understate their ages, and among old persons of both sexes to imagine themselves older than they really are. It might seem at first that, if the misstatements in the census and in the death register were the same, they would balance each

other, but a moment's reflection will show that this is not the case. As an example, suppose that the whole of the persons aged 65–75 returned themselves in the census as ten years older than they were, and that the ages of those who died between 65 and 75 were also returned ten years too high. Then the mortality deduced for this group of persons would appear as the mortality at ages 75–85, whereas it would in reality be the mortality at ages 65–75. There is one other age group in which obvious errors in the returns have necessitated special methods of correction, namely, the group of children under five years of age. In the official returns the populations and deaths of these children are shown at each separate year of age; but at both censuses more of each sex were returned at age 2 than at age 1, and, in the case of females, more were returned at age 3 than at age 1. The amount of misstatement of age is, however, probably less in the death registers than in the census, and the effect of this on the death rates of old people is to make the very oldest appear more healthy, and those who are not quite so old less healthy, than they really are. Fortunately, as far as their effect on the most important part of the life table is concerned, these errors tend to balance one another. The understating of the ages of women between 35 and 45 years of age would appear to be more serious. Dr. Farr* has, however, shown that (assuming that persons *over* 35 are less likely to be returned as *under* 35 in the death register than in the census) the error from this cause is less than might have been expected. His argument may be summarised thus:—If the group 25–35 in the census were increased by the addition of a number of persons who were really over 35, and if the same group in the death returns were increased by the deaths *among the same number of persons* over 35, the mortality at 25–35 would be made to appear too high; but, owing to the tendency already referred to, the addition to the group 25–35 in the death returns will be somewhat *less* than this, and therefore the death-rate will be *less disturbed* than at first seemed probable.

III.—Principles and methods of Life Table construction.

Before proceeding to show how rates of mortality for the *successive years* of life may be deduced from the figures for *groups of years* in table A, it will be well to indicate the way in which such rates may be used to form a life table. Suppose it to be ascertained that of 1,000 children born, 200 die in their first year of life, that of 1,000 children one year of age 90 die before completing their second year, and that of 1,000 aged two years 40 fail to complete their third year, the construction of the table would begin as follows:—(1) Of 100,000 born, 20,000 die within a year and 80,000 *survive at age one.* (2) Since 90 out of 1,000 die in their second year, the deaths in that year out of the 80,000 survivors will be $80{,}000 \times \frac{90}{1000} = 7{,}200$, and the survivors at age 2 will be $80{,}000 - 7{,}200 = 72{,}800$. (3) Since 40 out of 1,000 die in their third year, the deaths in that year out of the 72,800

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* English Life Table, No. 3, pp. xxi-xxii.

survivors will be $72{,}800 \times \frac{40}{1000} = 2{,}912$, and the survivors at age 3 will be $72{,}800 - 2{,}912 = 69{,}888$. If the results be entered in tabular form as they are obtained the table will begin thus:—

Age (x)	Dying in the year following x	Born, and Surviving at age x
0	20,000	100,000
1	7,200	80,000
2	2,912	72,800
3	...	69,888

Next, the number of deaths in a year out of 69,888 living at age 3 will be calculated and entered against age 3 in the death column, and the survivors at age 4 will be found by subtraction and inserted in the survivorship column, and so on. It will be seen that, since the table depends only on the *proportion* who die in each year, it is immaterial whether our rate of 90 per 1,000 for children aged one year is deduced from the deaths among 10,000 or 20,000 children. If there is an *excess* of children at age 1 in the population, there will be a *corresponding excess* of deaths; while if there are few children of that age, the deaths among them will be few *in the same proportion.* Thus, a life table *eliminates the effect of abnormal age distribution*, and (provided the numbers are not too small to give trustworthy rates) accurately sums up the effects of life conditions, whatever be the actual "age-constitution" of the population.

In the short specimen of life-table construction given above we have supposed ourselves to know how many out of 1,000 living at one birthday will survive till the next; but the census deals with ages in groups only, and tells us nothing about the numbers of persons who reach the various birthdays. We shall show, however, that the data at our disposal can readily be translated into the required form.

Suppose we have estimated from the census returns that the *average* number of persons living in a given period between 20 and 21 years of age was P, and suppose the death registers show that the *average annual* deaths in the same limits of age during the period were *d*. Now we may assume that the average age of persons between 20 and 21 years old is approximately 20½ years, and also that, of the deaths occurring among such persons, about half occur between the ages 20 and 20½, and half between 20½ and 21. The observed mortality, then (viz.: *d* deaths in a year among P persons between 20 and 21 years of age), is *consistent with the assumption* that in $P+\frac{d}{2}$ persons living at exactly 20 years of age, $\frac{d}{2}$ would die before reaching 20½, and $\frac{d}{2}$ more between 20½ and 21; that is to say, that of $P+\frac{d}{2}$ persons aged 20 years, $P-\frac{d}{2}$ would live to be 21, the number who would die in the interval being $\frac{d}{2}\times 2 = d$. This gives the *chance of living* from age 20 to age 21 as $\frac{P-\frac{d}{2}}{P+\frac{d}{2}}$ and the chance of dying as $\frac{d}{P+\frac{d}{2}}$. The assumption on which these

formulæ are based (viz., that if we trace a large number of persons from the age 20 to the age 21, those who die will, on the whole, die at equal intervals between the two ages) is not exactly true, but it is near enough to the truth for our purpose. Before we can use the formula $\frac{P-\frac{d}{2}}{P+\frac{d}{2}}$ we must, however, divide up the age-groups in Table A into single years, or else we must deduce some other figures belonging in a definite way to single years.* The question will be simplified by forming two new columns, as under :—

TABLE B.

Sex and age	Estimated mean Population, 1881-90		Deaths, 1881-90	
	In each age-group	At each age and upwards	In each age-group	At each age and upwards
MALES.				
Under 5 years...	35,835·5	267,353·7	30,089	69,692
5— ...	31,434·3	231,518·2	2,396	39,603
10— ...	29,003·4	200,083·9	1,075	37,207
15— ...	26,735·1	171,080·5	1,456	36,132
20— ...	25,464·3	144,345·4	1,771	34,676
25— ...	44,452·4	118,881·1	4,901	32,905
35— ...	33,393·4	74,428·7	6,528	28,004
45— ...	22,042·6	41,035·3	6,865	21,476
55— ...	12,429·4	18,992·7	6,762	14,611
65— ...	5,296·4	6,563·3	5,437	7,849
75— ...	1,184·2	1,266·9	2,158	2,412
85— ...	81·0	82·7	253	254
95 and up	1·7	1·7	1	1
FEMALES.				
Under 5 years...	36,527·8	291,747·3	25,859	65,890
5— ...	32,061·6	255,219·5	2,373	40,031
10— ...	29,525·0	223,157·9	1,089	37,658
15— ...	29,268·9	193,632·9	1,437	36,569
20— ...	29,633·5	164,364·0	1,810	35,132
25— ...	48,593·2	134,730·5	4,615	33,322
35— ...	35,977·0	86,137·3	5,557	28,707
45— ...	25,340·1	50,160·3	6,159	23,150
55— ...	15,436·2	24,820·2	7,042	16,991
65— ...	7,286·8	9,384·0	6,406	9,949
75— ...	1,902·1	2,097·2	3,039	3,543
85— ...	186·7	195·1	475	504
95 and up	8·4	8·4	29	29

* In some previous life tables it has been assumed that the mortality in any age group is equal to the mortality at the *middle age* of that group; for instance, that the mortality in the age group 35-45 may be taken as the mortality at the exact age 40. In this way a figure assumed to belong to a definite age is obtained, but by a somewhat questionable assumption. It may be that the mortality in the group 35-45 is just equal to that at age 39, or at 39½, or at 41; it is, therefore, unsafe to assume that it is the mortality *at 40*.

It may not be clear at once what is meant by "figures belonging in a definite way to single years." It may seem that the figures in Table A against ages 25, 35, &c., do belong in a definite way to those ages, at least as much as the figures "at each age and upwards" in Table B. Suppose, however, that we took the deaths at ages above 25 in Table A, and by some process of calculation inserted figures corresponding to them for each of the intermediate ages, we should obtain—

Deaths at age	25	and under	35
,,	,, 26	,,	36
,,	,, 27	,,	37
	&c.,		&c.

This would be nothing but an overlapping set of groups of 10 years; and we should be no nearer than before to our object of obtaining figures for *single years.*

But, if we deal with all ages above 25, all above 35, &c., and then insert intermediate values, we shall have—

All above	25
,,	26
,,	27
&c.,	&c.

And if we subtract "all above 26" from "all above 25," the remainder is the number between 25 and 26.

The new columns (formed by successively adding the figures in the old columns, beginning at the bottom) are simply a new way of stating our facts. We now have two columns of figures which by their very nature diminish from the top to the bottom. And if we can interpolate the corresponding figures at the separate years 1, 2, 3, 4, &c., we shall still have two diminishing sets of figures. But we cannot effect this interpolation without using some hypothesis as to the relations of the figures to the various ages to which they belong. Take for instance the deaths of males between 35 and 45. In the ten years the number of males who died at age 35 and upwards was 28,004, at 45 and upwards, 21,476. Let us try to deduce the probable number who died at age 40 years and upwards. Clearly it was something *less* than 28,004 (for it is included in that number) and *greater* than 21,476 (which is included in it). The simplest possible assumption is that the number is *half way between*, viz.:— $\frac{28,004 + 21,476}{2}$, or 24,740. The same assumption, applied to the next age-group, would give us $\frac{21,476 + 14,611}{2}$, or 18,043, for the number at age 50 and upwards. Carrying this a little further we should get the following figures:—

TABLE C.

Age	Deaths	
	At each age and upwards	In each age group of 5 years.
35	28,004	3,264
40	24,740	3,264
45	21,476	3,433
50	18,043	3,432
55	14,611	3,381
60	11,230	3,381
65	7,849	2,719
70	5,130	2,718
75	2,412	1,079
80	1,333	1,079
85	254	

Such a method as this (the forming of a set of arithmetical progressions, keeping at a constant difference for 10 years, then suddenly changing for the next ten years, and so on) is clearly inadequate. We can, however, escape from this objection by forming a *modified arithmetical progression*. As a simple example take the period from 65 to 85. We have :—

Deaths at 65 and upwards = 7,849
,, 75 ,, = 2,412
,, 85 ,, = 254

By certain processes of calculation we can produce the following series :—

TABLE D.

	Deaths at each age and upwards	Deaths in each age group	Successive Differences of Deaths in each group
65—	7,849	3,128	819
70—	4,721	2,309	820
75—	2,412	1,489	820
80—	923	669	...
85—	254	...	...

Here the deaths in the 5-year groups change *steadily*, instead of *irregularly* as before, and on the same principle figures can be calculated for each single year—66, 67, &c. Moreover, we can take four, five, or more of the original facts into account, and obtain a *modified arithmetical progression* running right through them in such a way that the numbers change steadily from one end to the other.

We have said that each interpolation involves some assumption as to the relation between the age and the number belonging to it. The short interpolation given above involves the assumption that the number of deaths at any age (x) and upwards $= 16 \cdot 395x^2 - 2839\,x + 123115 \cdot 125$. For, if in this formula we put $x = 65$, the whole expression reduces to 7849; if we put $x = 75$, it reduces to 2412; and if we put $x = 85$, it reduces to 254; and since it so far agrees with the facts on which this section of the table is based, there appears to be some ground for assuming that by putting $x = 66$, 67, &c., we shall get approximately true results for these intermediate ages. There is, however, a serious objection to the use of arithmetical (or "modified" arithmetical) progressions for figures relating to population or deaths. It is clear that a diminishing arithmetical progression (that is, a set of numbers decreasing by a constant difference) will presently result in *minus* numbers, which have no meaning in relation to population or deaths; and a modified arithmetical progression will end either in a series of *minus* numbers, or in a rapidly *increasing* series of *plus* numbers. It is true that such impossible results can only occur at the extreme end of a series, but the fact that they *may* occur is a strong argument against using this particular form of assumption. The example we have given above will make this plainer. Suppose we attempt to carry on that series beyond age 85. The deaths in each age-group diminish by 820 every five years; the deaths between 80 and 85 are 669, therefore those between 85 and 90 would (on this assumption) be $669 - 820 = -151$, or 151 less than nothing. Such a series is therefore useless to help us to estimate the numbers beyond the highest age for which facts are available.

We must then seek some other form of assumption. Returning to the same basis as before—

Deaths at	65	and upwards	7,849
,,	75	,,	2,412
,,	85	,,	 254

let us try first the effect of what is called *geometrical progression.* If x be such a number that $\frac{7{,}849}{x} = \frac{x}{2{,}412}$, then x is called the *geometrical mean* between 7,849 and 2,412, and if several numbers be inserted between 7,849 and 2,412, so that the proportion of each to the one preceding it shall be the same throughout, the whole forms a *geometrical progression.* The principle will be sufficiently illustrated by inserting the geometrical mean between 7,849 and 2,412 and assuming this to be the number of deaths at 70 and upwards; and, in the same way, assuming the geometrical mean between 2,412 and 254 to be the number at 80 and upwards. The series now becomes—

TABLE E.

Age	Deaths at each age and upwards	Deaths in each age-group
65—	7,849	3,498
70—	4,351	1,939
75—	2,412	1,629
80—	783	529
85—	254	...

These numbers are irregular, though less so than those in Table C; but it is evident that, as the principle of the method is that each number is to be some *proportion* of the number before it, we can never be confronted with *minus* numbers; however large or however small a proportion we take the result must always be *plus*.

Now if a set of numbers be in *geometrical* progression (*i.e.*, if they are successively in the same *proportion* to each other) their logarithms are in *arithmetical* progression (*i.e.*, they have a constant *difference*), and therefore a modified arithmetical progression of logarithms will correspond to a modified geometrical progression of the numbers they represent. The logarithms (or "logs.") of 7849, 2412, and 254 are—

3·8948143
3·3823773
2·4048337

and, applying to these the same methods of calculation as we used for Table D, we get—

TABLE F.

Age	Logs. of deaths at each age and upwards	Differences of logs.	Second difference
65—	3·8948143	—·1980802	—·1162766
70—	3·6967341	—·3143568	—·1162767
75—	3·3823773	—·4306335	—·1162766
80—	2·9517438	—·5469101	
85—	2·4048337		

and, finally, translating the column of logarithms back into numbers, and taking their differences—

TABLE G.

Age	Deaths at each age and upwards	Deaths in each age-group	Differences of deaths in age-groups
65	7,849	2,875	313
70	4,974	2,562	1,045
75	2,412	1,517	876
80	895	641	
85	254		

A hasty glance at the second and third columns might lead to the conclusion that this series is a very irregular one, whereas it is really remarkably regular; for its first column may be interpreted thus :—

Of the deaths at 65 and upwards 63 per cent. occur at 70 and upwards
" " 70 " 48 " 75 "
" " 75 " 37 " 80 "
" " 80 " 28 " 85 "

But let us see how the series will bear the test of being carried on beyond 85 years of age. Assuming the "second difference" to remain constant, the "difference of logs." at age 85 will be $-\cdot5469101 - \cdot1162766 = -\cdot6631867$.

TABLE H.

Age	Logs.	Differences of Logs.	Second difference of Logs.
85—	2·4048337	−·6631867	−·1162766
90—	1·7416470	−·7794633	−·1162766
95—	0·9621837	−·8957399	−·1162766
100—	0·0664438	−1·0120165	
105—	$\bar{1}$·0544273		

and, translating the logs. into numbers, the series of deaths will become :—

TABLE I.

	Deaths at each age and upwards	Deaths in each age-group	Differences of deaths in age-groups
65	7,849*	2,875	313
70	4,974	2,562	1,045
75	2,412*	1,517	876
80	895	641	442
85	254*	199	153
90	55	46	38
95	9	8	7·1
100	1	0·9	...
105	0·1	...	...

* The whole series is built on these three numbers.

The regularity of the series now becomes apparent, and the fact that it bears this test so well points to "modified geometrical progression" (or modified arithmetical progression of *logarithms*) as the form our calculations should take. For reasons such as have been here indicated logarithms are generally employed in interpolations connected with life table construction.

The principles here applied to insert *one* intermediate value in each interval of 10 years are equally applicable to the problem of inserting a value for *each year*. Thus, taking the logarithms at ages 65 and 75, and breaking up the difference into ten equal parts, we get an arithmetical progression of logarithms (corresponding to a geometrical progression of numbers) for each year from 65 to 75, the *differences* of this series of logarithms being *equal*. If we take in an adjoining period of life (55 to 65, or 75 to 85) we can make another series with another set of equal differences; or if we modify the differences through the whole 20 years, so that they shall increase or decrease by equal amounts from one end to the other, these "equal amounts" form a series of differences *of* differences, and the whole modified series is called *a series of two orders of differences*. Similarly, if we add a *third* 10 years, we can further modify the differences so that they shall proceed gradually through the 30 years. In this case the *second differences* will change by equal amounts, and we have *a series of three orders of differences*, and so on, adding a *fresh order of differences* for each extra 10-year period. We might in this way build up one great series of eight or nine differences through the whole of life; but this would involve very laborious arithmetical work, and we have therefore limited ourselves to series of four orders of differences, extending over periods of 40 years. Now a series constructed in this way for the 40 years 45-85, will be most nearly correct at and near its *middle age*, 65; for the figures we calculate for ages 64 and 66 will be almost equally influenced by the ascertained facts on both sides; whilst the figures we deduce at age 46 are not influenced at all by the ascertained facts at age 35, but only by those at ages 45, 55, 65, 75, and 85. And if we place a series 5-45 *end to end* with a series 45-85, the 10 years or so at the end of the first and the 10 years or so at the beginning of the second will not only be of doubtful value, but will very probably show the same kind of sudden change at the point of junction as we found at the junction of an ordinary series, 65-75, and another, 75-85. We therefore employ a number of series which *overlap* one another, and *weld* them together, in every case giving the *greatest weight* to the series which is nearest its middle point, and the *least weight* to the series which is just entering or just leaving. Thus we pass gradually from one series to another, and our final result shows no sudden or violent changes.

We now return to Table B, from which, on the principles laid down, the life table is to be deduced. From the population and deaths *at each age and upwards*, as given in that table, we can, by the method of differences, produce a similar series for each single year of age, and by subtracting the number at

and above any age from the number at and above any earlier age we get the number between the two ages, so that we can find the number living (P) and the deaths (d) *in each year of life*, and by the formula $p=\frac{P-\frac{d}{2}}{P+\frac{d}{2}}$ we can deduce the chance of *living through* that year of life. Or there is an alternative method. From table B we can readily, and without working out the *whole series*, deduce the number living at age 25 and under 26, considered as the centre point of the series 5–45; at age 35 and under 36, considered as the centre point of the series 15–55, and so on; and, having treated the deaths in the same way, we can find the chances of living a year from age 25, a year from age 35, &c., and finally work out a series of these chances instead of one series of populations and another of deaths; or we can still more readily estimate the chance of living from 24½ to 25½, from 34½ to 35½, &c.,* and from a set of these chances, at intervals of 10 years, we can form a series giving the corresponding chances for the successive years.

This method of interpolation is applicable to all ages from 5 years onwards; but, owing to the rapid changes in the rate of mortality in the earliest years of life, it is necessary to use the populations and deaths at *separate ages* in calculating the chances of survival under 5 years of age. We have already pointed out that the returns of population at these ages are untrustworthy; the method used for correcting them is as follows:—The deaths under one year of age in the 10 years 1881–90 include the *whole* of the mortality, under one year of age, of children born in the nine years 1881–89, and *part* of that of children born in 1880 and in 1890; the deaths under one in the ten years may therefore be fairly taken as occurring out of ½ births in 1880 + births in 1881–89 + ½ births in 1890. Similarly the deaths under one year in the ten years 1880–89 may be taken as occurring out of ½ births in 1879 + births in 1880–88 + ½ births in 1889; and subtracting these deaths from the aggregate births out of which they occurred we get the children aged one year out of whom the deaths between 1 and 2 years of age in 1881–90 occurred. Continuing the same process, we estimate the survivors in 1881–90 at ages 2, 3, and 4, out of whom the deaths at 2–3, 3–4, and 4–5 occurred. This process gives—

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* The following statement will make the formula for this process clear to those acquainted with the elements of the differential calculus:—

If $\phi(x)$ be the number living at age x and upwards, and $\psi(x)$ be the deaths at age x and upwards, $\phi(x)-\phi(x+h)$ is the number living in the interval between age x and age $x+h$, $\psi(x)-\psi(x+h)$ is the number of deaths between age x and age $x+h$, and $\frac{\psi(x)-\psi(x+h)}{\phi(x)-\phi(x+h)}$ is the rate of mortality in this interval. Now $\frac{\psi(x)-\psi(x+h)}{\phi(x)-\phi(x+h)}$ may be written $\frac{\frac{\psi(x)-\psi(x+h)}{h}}{\frac{\phi(x)-\phi(x+h)}{h}}$ which becomes $\frac{\psi'(x)}{\phi'(x)}$ when x is indefinitely diminished.

So $\frac{\psi'(x)}{\phi(x)}$ is the rate of mortality ($=m$) at the exact age x, and $p=\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$.

	Males	Females
At birth	97,430	94,679
At age 1	79,638	80,409
,, 2	73,378	74,623
,, 3	71,088	72,408
,, 4	69,776	70,950
Totals ..	391,310	393,069

as the numbers out of which the recorded deaths in the ten years at the various ages occurred. But, on the basis of the recorded deaths and our estimate of the mean population under 5 years of age, the totals should be 373,400 males and 378,208 females. Assuming these figures to be correct, dividing them out in the proportions just determined, and adding columns showing the recorded deaths, we get—

Age	Number at each age out of whom the deaths occurred*		Number of deaths during 1881-90 in each year of age	
	Males	Females	Males	Females
Birth	92,971	91,099	17,990	14,422
Age 1	75,993	77,369	6,751	6,100
,, 2	70,020	71,802	2,579	2,556
,, 3	67,834	69,670	1,640	1,602
,, 4	66,582	68,268	1,129	1,179
Totals	373,400	378,208	30,089	25,859

From this table the several chances of living 1 year can be directly calculated, the figures at ages 3 and 4 being afterwards used in conjunction with the estimates already made for ages 5, 15, &c., to determine the chance of living a year at age 5. Finally, the interpolation having given us the chance of living from 24½ to 25½, and from 25½ to 26½, the chance of living from 25 to 26 is taken as the *geometrical mean* of these two chances, and similarly for all the ages above 5. Thus, finally, we have the successive chances (p_x) of living 1 year, from birth to old age, for each sex These figures are shown in columns 1 and 2 of Table 3 (pp. 32–33).

* These numbers are the aggregates in the 10 years; the mean annual numbers are obtained by dividing them by 10.

We can now bring our results into line with those of the imaginary ledger with which we commenced this sketch. According to the experience of Manchester in 1881–90, the 100,000 children born would have consisted of 50,764 males, and 49,236 females. If the rates of mortality to which these were exposed through life had been the same as those in Manchester in 1881–90, 40,941 males and 41,441 females would have reached their first birthday, 9,823 males and 7,795 females having died in the interval; 37,304 males and 38,174 females would have lived to the end of the second year, and so on. The survivors (l_x) at each year of age are shown in column 2, and the deaths (d_x) in column 1 of Tables 1 and 2 (pp. 28–31). The mean number living, or the years of life lived (P_x),* in each year of age is shown in column 3; and the sum of all the numbers living at age x and upwards (Q_x), which is also the *total years of life* lived from age x upwards, in column 4. The expectation of life at each age (E_x), or the average future years of life to each of the l_x who reach age x, completes Tables 1 and 2.

Table 3 gives, in columns 3 and 4, the survivors at each year of 100,000 males and 100,000 females born, and in its remaining columns a summary of the results in Tables 1 and 2.

A few examples will show how the tables may be read. Of the 50,764 males born, some die very shortly after birth, others at lengthening intervals through the first year, 40,941 surviving at the end of the year. If all lived through the year they would live 50,764 *years of life*, while if all the 9,823 deaths happened immediately after birth they would live only 40,941 *years of life*; but the deaths are so spread through the year that the years of life amount to 44,765. In the second year the 40,941 survivors at one year of age live 39,080 years of life, and 37,304 of them survive at age 2. These 37,304 live 36,611 years in the third year of life, and so on. In the *whole of life* the 50,764 born live 1,761,996 years (Q_0), or an average of 34·71 years (E_0) each; so the average at death of all who are born will be 34·71 years. The 40,941 who reach one year of age live 39,080 years of life in their next year, 36,611 in the following year, and so on. In their whole future life they live 1,717,231 years (Q_1), or an average of 41·94 (E_1) years each; so the average age at death of those who reach one year of age will be 1+41·94=42·94. The 30,278 who reach age 25 live 30,152, 29,891, &c., years of life in the following years, or 929,174 years in all, giving an average of 30·69 years of future lifetime each; the average age at death of those who reach 25 will therefore be 25+30·69=55·69 If we want to know how many years of life they live between age 25 and age 65 we can find it by adding up the corresponding 40 numbers in the P_x column, or more easily by subtracting

* With one exception P_x has been taken as the geometrical mean between l_x and l_{x+1}. The exception is in the first year of life; in this case the mean number living has been deduced from the recorded proportions of deaths under 3 months, between 3 months and 6 months, and between 6 months and 1 year.

Q_{65} (=74,787) from Q_{25} (=929,174). The number is therefore 854,387, or an average of 28·22 each, showing that of their 30·69 years of future lifetime 28·22 years will be under, and 2·47 years over, the age of 65.

IV.—Practical utility of Life Tables.

Before discussing in fuller detail certain of the uses to which a life table may be put, let us inquire in what respects our Manchester Table differs from the imaginary table on which it is modelled as to form. In this *imaginary table* we begin with a number of infants born in a particular place, and we record their length of life and their deaths, wherever their lives may be spent or their deaths occur, and whatever may be the varying influences of social and sanitary changes to which they may be subjected in the course of years. In the case of our *actual life table* we consider the vital conditions of groups of persons who, wherever they were born, and whatever may have been the varying conditions of their lives, were, at all events, inhabitants of Manchester at some time in the ten years 1881–90. The rates of mortality which we deduce represent the combined effect of the past history of these people and of the present health conditions of Manchester. We cannot possibly disentangle the two elements, for we have no data that will enable us to distinguish between the death-rate of men aged 45 who have lived all their lives in Manchester, and the death rate of men aged 45 who have lived there only one year, two years, &c. Our table is therefore the vital history in 1881-90 of the people who then formed the population of Manchester.

But, it may fairly be asked, is the result of these elaborate calculations of any practical use? Is it anything more than an arithmetical curiosity? Most assuredly. The work might be defined as the "vital history of Manchester in the past decennium"; but this definition is inadequate, for history may be made to invest trivial events with a factitious importance, at the expense of other events which are of real moment. The life table is the vital history so written that each of the facts with which it deals has its due place, and is shown in due relationship to other facts.

Mr. Noel A. Humphreys in his life table for England and Wales, 1876-80,* set himself to answer the question "are we young longer, or mature longer, or old longer?" We may ask the question "Are people in Manchester young, mature, or old, for longer or shorter periods than people in England and Wales as a whole?" No life table for the whole country for years subsequent to 1880 has yet appeared, but while waiting for such a table it will be useful to institute a comparison between the life table for Manchester in 1881-90 and Dr. Farr's life table for England and Wales in 1838-54, or Dr. Ogle's for the period 1871-80. The following short table shows the practical nature of such a comparison :—

* Journal of the Statistical Society, Vol. xlvi. (1883), pp. 189—213.

TABLE J.—OF 100,000 CHILDREN BORN, THE NUMBERS SURVIVING AT VARIOUS AGES.

Age	PERSONS		MALES			FEMALES		
	According to the experience of		According to the experience of			According to the experience of		
	England and Wales	Manchester	England and Wales		Manchester	England and Wales		Manchester
	1838-54	1881-90	1838-54	1871-80	1881-90	1838-54	1871-80	1881-90
0	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
5	73,682	69,814	72,372	73,407	67,896	75,055	76,262	71,792
15	68,456	64,818	67,278	69,642	63,076	69,692	72,496	66,614
25	63,405	61,444	62,422	65,708	59,645	64,434	68,486	63,300
45	50,292	46,386	49,577	52,237	43,664	51,040	56,017	49,192
65	30,903	21,002	29,459	29,716	18,067	32,417	35,617	24,027

These figures are in themselves a sufficient answer to the question whether a life table is of any use. The experience of England and Wales in 1871-80 shows 65,708 males, out of 100,000 born, entering the period of life 25-65, which may be taken to represent roughly the period of most men's best and most useful work, and 29,716 of these surviving through the whole period, 35,992 dying in the interval: while the experience of Manchester in

1881-90 shows only 59,645 out of a like number of births entering the same life period, and only 18,067 surviving through it, as many as 41,578 dying in the interval. And if we look a little more closely into the figures their lesson is driven home with relentless force. Let us try to explain the fact that fewer children in Manchester reach maturity, by the argument which has sometimes been used, that the weaklings have died off. Then our 59,645 in Manchester at age 25 should be at least as well fitted for the battle of life as the 65,708 in England and Wales, and we might look for at least an equal proportion to survive till age 65; but if out of 65,708 at 25 years of age 29,716 survive till 65, then the survivors out of 59,645 should be 26,974, whereas our life table shows them as 18,067 only. Why have the 8,907 died who should have survived? This is an eminently practical question. Did they start life with crippled vitality? Or were the conditions of childhood such that their vitality had become impaired before they reached manhood? Or was the battle of life more severe for them than for others, or the conditions under which they had to fight it less favourable? A closer examination of the figures, as in the following table, throws a suggestive light on this question:—

TABLE K.

OF 1,000 ENTERING EACH LIFE-PERIOD, THE NUMBER SURVIVING THROUGH THE PERIOD.

Life-period	Persons: England and Wales, 1838-54	Persons: Manchester, 1881-90	Males: England and Wales, 1838-54	Males: England and Wales, 1871-80	Males: Manchester, 1881-90	Females: England and Wales, 1838-54	Females: England and Wales, 1871-80	Females: Manchester, 1881-90
0–5	737	698	724	734	679	751	763	718
5–15	929	928	930	949	929	929	951	928
15–25	926	948	928	944	946	925	945	950
25–45	793	755	794	795	732	792	818	777
45–65	614	453	594	569	414	635	636	488

The figures in this table lend some colour to the suggestion that the effect of the greater mortality among young children is to weed out the more feeble, so that those who reach maturity shall be "selected lives." For, while the proportion reaching their fifth year of life in Manchester is much below that in England and Wales in either period, the proportion of those aged 5 who live to be 15 is practically as great in Manchester as it was in the whole country in 1838-54, and not far below the proportion of 1871–80; while the chance of living from 15 to 25 in Manchester is actually greater than in England and Wales in either period. Unfortunately for the hypothesis, there is another and much more probable explanation. In the ten years, 1881-90, the births in the three districts exceeded the deaths by 56,355, while the actual increase of population was nearly 66,000. These figures point to the existence of a stream of immigrants from outside into Manchester; and it is at least probable that many of these are young adults from rural districts enjoying

TABLE I.—EXPECTATIONS OF LIFE AT VARIOUS AGES.

Ages	Persons		Males			Females		
	According to the experience of		According to the experience of			According to the experience of		
	England and Wales	Manchester	England and Wales		Manchester	England and Wales		Manchester
	1838-54	1881-90	1838-54	1871-80	1881-90	1838-54	1871-80	1881-90
0	40·86	36·55	39·91	41·35	34·71	41·85	44·62	38·44
5	50·02	46·84	49·71	50·87	45·59	50·33	53·08	48·06
15	43·54	40·15	43·18	43·41	38·78	43·90	45·63	41·50
25	36·57	32·05	36·12	35·68	30·69	37·04	37·98	33·38
45	23·41	18·84	22·76	22·07	17·80	24·06	24·06	19·79
65	11·17	8·69	10·82	10·55	8·15	11·51	11·42	9·11

more robust health than the natives of Manchester of similar ages, and whose immigration would undoubtedly reduce the death-rate at the ages affected. It may be that each of the suggested explanations has something to do with the comparatively low mortality in Manchester between 15 and 25 years of

age; but after the age 25 neither the selection which may possibly result from the early weeding out of unsound lives, nor the improvement brought about by the introduction of healthier lives from outside, avails to prevent a rapidly increasing mortality. Thus, while in England and Wales, of 1,000 men aged 25, nearly 800 survive to be 45, and of 1,000 aged 45, 569 survive to be 65, in Manchester the survivors are only 732 and 414 respectively. A further important question grows out of these figures:—If the power of living diminishes more rapidly in one population than in another, is it not probable that the capacity for useful work concurrently diminishes more rapidly among those who survive?

So far, our comparison of the life tables for England and Wales with the table for Manchester has been based on the survivors at various ages. In table L the expectations of life, or average future lifetimes, at these ages are compared.

It will be seen that the expectations of life in Manchester are, at all these ages, considerably less than in England and Wales, and if we calculate the *percentages* of the Manchester figures to the English figures, the progressive deterioration in Manchester as age advances will become apparent.

TABLE M.

THE PROPORTIONS PER CENT. WHICH THE EXPECTATIONS OF LIFE IN MANCHESTER BEAR TO THOSE IN ENGLAND AND WALES.

Age	Expectations of life in Manchester, 1881-90, compared with those in England and Wales, taken as 100.				
	Persons	Males		Females	
	1838-54	1838-54	1871-80	1838-54	1871-80
0	89	87	84	92	86
5	94	92	90	95	91
15	92	90	89	95	91
25	88	85	86	90	88
45	80	78	81	82	82
65	78	75	77	79	80

Thus, comparing males in Manchester in 1881-90 with males in England and Wales in 1871–80, at birth the expectation of life is only 84 per cent., at 5 years of age it has risen to 90 per cent., at 15 years of age it is 89 per cent., and it steadily decreases at each subsequent period.

The above two tables deal with the whole future lifetime at each age. Mr. Humphreys, in the paper already alluded to, divided the whole of lifetime into three periods, in order to show what proportions of the total

lifetime were passed in youth, in maturity, and in old age. Adopting the principle of this method, but dividing the whole of life into six periods instead of three, we get the following results:—

TABLE N.

THE EXPECTATION OF LIFE IN ENGLAND AND WALES, 1838–54, AND IN MANCHESTER, 1881–90, DIVIDED INTO SIX LIFE-PERIODS.

Life Period	Age—Limits of Period	Length of Period in Years	Persons		Males		Females	
			England and Wales	Manchester	England and Wales	Manchester	England and Wales	Manchester
The whole of life ...	All ages	(?)	40·86	36·55	39·91	34·71	41·85	38·44
Infancy	0–5	5	4·00	3·85	3·94	3·76	4·07	3·94
School age ..	5–15	10	7·05	6·67	6·92	6·50	7·19	6·86
Adolescence.	15–25	10	6·62	6·33	6·51	6 15	6·73	6·51
Maturity ...	25–45	20	11·42	10·96	11·26	10·53	11·58	11·39
	45–65	20	8·32	6 91	8·09	6·30	8·55	7·55
Decline ...	65 and upwards	(?)	3·45	1·83	3·19	1·47	3·73	2·19

NOTE.—The Table may be read thus:—According to Manchester experience, males live 34·71 years each, on the average; so 100 males born live 3471 years of life in the aggregate; and of these 3471 years of life, 376 are lived at ages under 5 years, 650 between 5 and 15 years of age, 615 between 15 and 25 years of age, and so on.

Here, it would seem, we have a complete answer to the question we proposed, "Are people in Manchester young, mature, or old for longer or shorter periods than in England and Wales as a whole?" The answer must, however, be accepted with caution. If we take the three periods under 25 years of age to represent youth, the two periods between 25 and 65 to represent maturity, and ages above 65 to represent old age, it will be found that males in Manchester are young for 94 per cent., mature for 87 per cent., and old for 46 per cent. as long as in England and Wales. And if we take one and the same age as marking the transition from maturity to decline in the two cases, we shall be tempted to conclude that the comparison is not wholly unfavourable to Manchester; for while its inhabitants lose more than half of their old age, they lose only 13 per cent. of their maturity, and only 6 per cent. of their youth. But may we make such an assumption? It has already been suggested in the course of these remarks that a diminished power of living is accompanied by a diminished capacity for work in those who survive: and when we find that the expectation of life of men aged 65 in Manchester is only 8·15 years, while in England and Wales it is over 10½ years (see Table L), we are almost forced to the conclusion that in Manchester men *grow old sooner* than in the country as a whole.

Before passing from this subject it may be useful to indicate the direction in which a true comparison of the working powers of different communities may some day be made. The life table gives us the numbers (out of a certain number born) who reach and live through each year of life. If, now, the average working power of a man at each age could be expressed in terms of the *vitality* at that age, or in terms of the *expectation of life* at that age; that is, if some numerical relation could be established between the power of *living* and the power of *working*; then, by multiplying the *number living* by the *average working power*, at any age, we should obtain the total working power of all living at that age. Thus we could add a new column to the life table, and the sum of all the numbers in that column would give us the total working power through life of all the children born.

In the absence of such data, the following illustration indicates a more exact method of comparison than that given above. In the English life table (1838-54) the expectation of life for males at the age of 65, which we have taken as the boundary between the periods of maturity and decline, is 10·82 years, and the chance of living one year from the same age is ·95409. In the Manchester Life Table 10·82 is about half-way between the expectations of life at 58 and 59 (see Table 1), and may therefore be taken as the expectation at age 58½, while ·95409 is slightly above the chance of living a year at age 57. That is to say, the vitality of men at 57 in Manchester is about equal to that of men at 65 in England, and the expectation of life of men at 58½ in Manchester is equal to that of men at 65 in England. So, if we take 65 as the beginning of the period of decline in England, we should take 57 or 58½ as the beginning of the period of decline in Manchester. Let us take the latter age, as giving the most favourable result for Manchester, and analyse the expectations of life in England and in Manchester into periods of maturity and decline, the boundary between these periods being at age 65 in England and at age 58½ in Manchester.

TABLE O.

EXPECTATION OF LIFE OF MALES AGED 25, DIVIDED INTO PERIODS OF MATURITY AND DECLINE.

	England and Wales, 1838-54		Manchester, 1881-90	
	Expectation in Years	Per cent. of total Expectation	Expectation in Years	Per cent. of total Expectation
Maturity.........	31·01	86	25·75	84
Decline.........	5·11	14	4·94	16
	36·12	100	30·69	100

This table shows that the *total* expectation of life of men aged 25 in England is 36·12 years, of which 31·01 years, or 86 per cent., belong to the

period of maturity, and 5·11 years, or 14 per cent., to the period of decline; while the expectation in Manchester at the same age is only 30·69 years, of which 25·75 years, or 84 per cent., belong to the period of maturity, and 4·94 years, or 16 per cent., to the period of decline. Or, in terms similar to those of the question we proposed, men aged 25 in Manchester will lose $\frac{1}{6}$ of their normal period of maturity, and $\frac{1}{30}$ of their normal period of decline.

The life table suggests many questions, and itself supplies the answers to some of them. For the solution of others we must look to the causes from which men die; and these, when ranged in the framework which the table supplies, will trace for us the nature of the wave of mortality which prematurely submerges so many thousands. All this, however, is matter for careful and patient analysis, and cannot be dealt with in our present limits. The life table is the instrument which makes such analysis possible, and the analysis will in its turn indicate the directions in which the excessive mortality should be most resolutely combated. That there is an irreducible minimum—that a part, and perhaps a great part, of the excess is inseparable from life and work in a great manufacturing city—is possible and indeed probable; but it is the obvious duty of those who are charged with responsibility for the social and sanitary well-being of the people to refuse to admit that the mortality at any given time has reached this minimum.

We have sketched out one of the uses of a life table; another use almost suggests itself. It will be the duty of someone, ten years hence, to compile the vital history of Manchester in 1891–1900. The present life table will make the life table of 1891–1900 doubly valuable; for it will then be possible to sum up the value of sanitary work that is being done now. The two tables side by side will form a balance sheet of profit or loss in the second period as compared with the first. And in the meantime a valuable comparison can be made year by year; for the life table shows us the sex and age-constitution of a population which (with the death-rates of Manchester in 1881-90) would be constantly kept up to 3,654,527 by 100,000 births in each year. And, by means of the death-rates at the various ages in any year, we can approximately ascertain whether more or less than 100,000 births would be required to balance the effect of such death-rates. The life table is therefore an instrument by which to measure the true significance of the mortality year by year, and thus to show the direction of changes that may take place in the mortality at various ages. It is true that such comparison can be made, after a fashion, without the aid of a life table, by simply comparing the death-rates at various ages in one year with those in other years; and, in the same way, the death-rates in Manchester in any year may be compared with those in England and Wales in the same year. But only a life table can show the combined effect of a greater rate at one age and a smaller rate at another. And even if (as in our comparison of Manchester with all England) the rates are uniformly higher in one case than in the other, only a life table can show the effect thus produced on the total duration of life, and on the comparative amounts of childhood, maturity, and old age.

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881-1890.

Table 1	- - -	Males.
Table 2	- - -	Females.
Table 3	- - -	Males and Females.

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 1.—Males.

Columns	1	2	3	4	5
Age x	Dying in each Year of Age d_x	Born, and Surviving at each Age l_x	Population, or Years of Life lived, in each Year of Age P_x	Population, or Years of Life lived, in and above each Year of Age Q_x	Expectation of Life at each Age $E_x\left(=\frac{Q_x}{l_x}\right)$
0	9,823	50,764	44,765	1,761,996	34·71
1	3,637	40,941	39,080	1,717,231	41·94
2	1,374	37,304	36,611	1,678,151	44·99
3	869	35,390	35,493	1,641,540	45·69
4	594	35,061	34,763	1,606,047	45·81
5	446	34,467	34,243	1,571,284	45·59
6	375	34,021	33,833	1,537,041	45·18
7	317	33,646	33,487	1,503,208	44·68
8	268	33,329	33,195	1,469,721	44·10
9	229	33,061	32,946	1,436,526	43·45
10	198	32,832	32,733	1,403,580	42·75
11	175	32,634	32,546	1,370,847	42·01
12	156	32,459	32,381	1,338,301	41·23
13	145	32,303	32,230	1,305,920	40·43
14	138	32,158	32,089	1,273,690	39·61
15	136	32,020	31,952	1,241,601	38·78
16	137	31,884	31,816	1,209,649	37·94
17	143	31,747	31,676	1,177,833	37·10
18	150	31,604	31,529	1,146,157	36·27
19	161	31,454	31,373	1,114,628	35·44
20	173	31,293	31,206	1,083,255	34·62
21	187	31,120	31,026	1,052,049	33·81
22	202	30,933	30,832	1,021,023	33·01
23	218	30,731	30,622	990,191	32·22
24	235	30,513	30,395	959,569	31·45
25	252	30,278	30,152	929,174	30·69
26	269	30,026	29,891	899,022	29·94
27	286	29,757	29,613	869,131	29·21
28	304	29,471	29,318	839,518	28·49
29	321	29,167	29,006	810,200	27·78
30	338	28,846	28,677	781,194	27·08
31	354	28,508	28,331	752,517	26·40
32	371	28,154	27,968	724,186	25·72
33	387	27,783	27,589	696,218	25·06
34	403	27,396	27,194	668,629	24·41
35	419	26,993	26,783	641,435	23·76
36	434	26,574	26,357	614,652	23·13
37	449	26,140	25,915	588,295	22·51
38	464	25,691	25,458	562,380	21·89
39	478	25,227	24,987	536,922	21·28
40	492	24,749	24,502	511,935	20·68
41	504	24,257	24,004	487,433	20·09
42	518	23,753	23,492	463,429	19·51
43	529	23,235	22,969	439,937	18·93
44	540	22,706	22,434	416,968	18·36
45	552	22,166	21,888	394,534	17·80
46	561	21,614	21,332	372,646	17·24
47	572	21,053	20,765	351,314	16·69
48	583	20,481	20,187	330,549	16·14
49	592	19,898	19,600	310,362	15·60
50	604	19,306	19,002	290,762	15·06
51	614	18,702	18,392	271,760	14·53
52	627	18,088	17,772	253,368	14·01
53	638	17,461	17,139	235,596	13·49
54	650	16,823	16,495	218,457	12·99

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 1.—Males.—(*Continued.*)

Columns	1	2	3	4	5
Age	Dying in each Year of Age	Born, and Surviving at each Age	Population, or Years of Life lived, in each Year of Age	Population, or Years of Life lived, in and above each Year of Age	Expectation of Life at each Age
x	d_x	l_x	P_x	Q_x	$E_x\left(=\frac{Q_x}{l_x}\right)$
55	662	16,173	15,838	201,962	12·49
56	675	15,511	15,170	186,124	12·00
57	685	14,836	14,490	170,954	11·52
58	696	14,151	13,798	156,464	11·06
59	705	13,455	13,098	142,666	10·60
60	711	12,750	12,389	129,568	10·16
61	717	12,039	11,675	117,179	9·73
62	719	11,322	10,957	105,504	9·32
63	718	10,603	10,238	94,547	8·92
64	713	9,885	9,522	84,309	8·53
65	706	9,172	8,812	74,787	8·15
66	694	8,466	8,112	65,975	7·79
67	678	7.772	7,425	57,863	7·45
68	659	7,094	6,756	50,438	7·11
69	636	6,435	6,109	43,682	6·79
70	609	5,799	5,486	37,573	6·48
71	579	5,190	4,892	32,087	6·18
72	546	4,611	4,329	27,195	5·90
73	511	4,065	3,801	22,865	5·63
74	473	3,554	3,309	19,065	5·36
75	435	3,081	2,855	15,756	5·11
76	396	2,646	2,440	12,901	4·88
77	355	2,250	2,065	10,461	4·65
78	316	1,895	1,730	8,396	4·43
79	278	1,579	1,433	6,666	4·22
80	242	1,301	1,173	5,233	4·02
81	208	1,059	949	4,060	3·83
82	175	851	758	3,111	3·66
83	147	676	598	2,353	3·48
84	121	529	465	1,755	3·32
85	98	408	356	1,290	3·16
86	78	310	268	934	3·01
87	61	232	199	666	2·87
88	47	171	145	467	2·73
89	36	124	104	322	2·60
90	27	88	73	218	2·48
91	19	61	51	145	2·36
92	14	42	34	94	2·24
93	10	28	23	60	2·12
94	6	18	15	37	1·99
95	5	12	9	22	1·86
96	2	7	6	13	1·69
97	2	5	4	7	1·49
98	1	3	2	3	1·20
99	1	2	1	1	0·75
100	1	1	0	0	...
101	...	0	...	...	...
102	...	...	...	...	...
103	...	...	...	...	...
104	...	...	...	...	...
105	...	...	...	...	...

Note.—The figures at the higher ages in Columns 2, 3, and 4 were calculated to two places of decimals. For convenience the nearest whole numbers only are printed, but the expectations of life in Column 5 are derived from the more exact values.

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 2.—Females.

Columns	1	2	3	4	5
Age	Dying in each Year of Age	Born, and Surviving at each Age	Population, or Years of Life lived, in each Year of Age	Population, or Years of Life lived, in and above each Year of Age	Expectation of Life at each Age
x	d_x	l_x	P_x	Q_x	$E_x\left(=\frac{Q_x}{l_x}\right)$
0	7,795	49,236	44,551	1,892,531	38·44
1	3,267	41,441	39,774	1,847,980	44·59
2	1,359	38,174	37,488	1,808,206	47·37
3	846	36,815	36,389	1,770,718	48·10
4	622	35,969	35,657	1,734,329	48·22
5	482	35,347	35,105	1,698,672	48·06
6	403	34,865	34,663	1,663,567	47·71
7	336	34,462	34,294	1,628,904	47·27
8	282	34 126	33,985	1,594,610	46·73
9	237	33,844	33,725	1,560,625	46·11
10	203	33,607	33,505	1,526,900	45·43
11	175	33,404	33,316	1,493,395	44·71
12	156	33,229	33,151	1,460,079	43·94
13	141	33,073	33,002	1,426,928	43·14
14	134	32,932	32,865	1,393,926	42·33
15	130	32,798	32,733	1,361,061	41·50
16	130	32,668	32,603	1,328,328	40·65
17	135	32,538	32,471	1,295,725	39·82
18	141	32,403	32,333	1,263,254	38·99
19	151	32,262	32,186	1,230,921	38·15
20	162	32,111	32,030	1,198,735	37·33
21	175	31,949	31,862	1,166,705	36·52
22	188	31,774	31,680	1,134,843	35·72
23	202	31,586	31,485	1,103,163	34·93
24	218	31,384	31,275	1,071,678	34·15
25	232	31,166	31,050	1,040,403	33·38
26	247	30,934	30,811	1,009,353	32·63
27	261	30,687	30,556	978,542	31·89
28	276	30,426	30,287	947,986	31·16
29	290	30,150	30,004	917,699	30·44
30	303	29,860	29,708	887,695	29·73
31	315	29,557	29,399	857,987	29·03
32	327	29,242	29,078	828,588	28·34
33	337	28,915	28,746	799,510	27·65
34	348	28,578	28,403	770,764	26·97
35	358	28,230	28.050	742,361	26·30
36	368	27,872	27,687	714,311	25·63
37	376	27,504	27,315	686,624	24·96
38	386	27,128	26,934	659,309	24·30
39	395	26,742	26,544	632,375	23·65
40	404	26,347	26,144	605,831	22·99
41	414	25,943	25,735	579,687	22·34
42	425	25,529	25,316	553,952	21·70
43	436	25,104	24,885	528,636	21·06
44	448	24,668	24,443	503,751	20·42
45	461	24,220	23,989	479,308	19·79
46	475	23,759	23,520	455,319	19·16
47	491	23,284	23,037	431,799	18·54
48	508	22,793	22,538	408,762	17·93
49	524	22,285	22,022	386,224	17·33
50	542	21,761	21,488	364,202	16·74
51	560	21,219	20,937	342,714	16·15
52	578	20,659	20,368	321,777	15·58
53	597	20,081	19,780	301,409	15·01
54	615	19,484	19,174	281,629	14 45

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 2.—Females.—(*Continued.*)

Columns	1	2	3	4	5
Age	Dying in each Year of Age	Born, and Surviving at each Age	Population, or Years of Life lived, in each Year of Age	Population, or Years of Life lived, in and above each Year of Age	Expectation of Life at each Age
x	d_x	l_x	P_x	Q_x	$E_x\left(=\frac{Q_x}{l_x}\right)$
55	633	18,869	18,550	262,455	13·91
56	650	18,236	17,908	243,905	13·37
57	668	17,586	17,249	225,997	12·85
58	685	16,918	16,572	208,748	12·34
59	702	16,233	15,878	192,176	11·84
60	717	15,531	15,168	176,298	11·35
61	730	14,814	14,444	161,130	10·88
62	742	14,084	13,708	146,686	10·42
63	752	13,342	12,960	132,978	9·97
64	760	12,590	12,204	120,018	9·53
65	764	11,830	11,441	107,814	9·11
66	766	11,066	10,676	96,373	8·71
67	763	10,300	9,911	85,697	8·32
68	756	9,537	9,151	75,786	7·95
69	744	8,781	8,401	66,635	7·59
70	728	8,037	7,665	58,234	7·25
71	706	7,309	6,947	50,569	6·92
72	681	6,603	6,253	43,622	6·61
73	650	5,922	5,588	37,369	6·31
74	615	5,272	4,955	31,781	6·03
75	577	4,657	4,359	26,826	5·76
76	536	4,080	3,803	22,467	5·51
77	493	3,544	3,289	18,664	5·27
78	448	3,051	2,819	15,375	5·04
79	403	2,603	2,393	12,556	4·82
80	359	2,200	2,013	10,163	4·62
81	316	1,841	1,676	8,150	4·43
82	275	1,525	1,381	6,474	4·24
83	236	1,250	1,126	5,093	4·07
84	200	1,014	908	3,967	3·91
85	169	814	725	3,059	3·76
86	139	645	572	2,334	3·62
87	113	506	446	1,762	3·48
88	92	393	344	1,316	3·35
89	73	301	262	972	3·23
90	58	228	197	710	3·11
91	44	170	146	513	3·00
92	34	126	108	367	2·90
93	26	92	78	259	2·80
94	19	66	56	181	2·70
95	14	47	40	125	2·61
96	10	33	28	85	2·51
97	7	23	19	57	2·42
98	5	16	13	38	2·32
99	4	11	9	25	2·20
100	2	7	6	16	2·07
101	2	5	4	10	1·89
102	1	3	3	6	1·65
103	1	2	2	3	1·31
104	1	1	1	1	0·80
105	...	0	0	0	...

Note.—The figures at the higher ages in Columns 2, 3, and 4 were calculated to two places of decimals. For convenience the nearest whole numbers only are printed, but the expectations of life in Column 5 are derived from the more exact values.

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 3.—Males and Females.

Age x	Chance of Living one Year from each age p_x — Males	Chance of Living one Year from each age p_x — Females	Of 100,000 Males born, the Number surviving at each Age	Of 100,000 Females born, the number surviving at each Age	Of 100,000 of both Sexes (50,764 Males + 49,236 Females) born — The Number surviving at each Age l_x	Population, or Years of Life lived in each Year P_x	Population, or Years of Life lived in and above each Year Q_x
0	·80650	·84169	100.000	100,000	100,000	89,316	3,654,527
1	·91116	·92116	80,650	84,169	82,382	78,854	3,565,211
2	·96317	·96440	73,485	77,533	75,478	74,099	3,486,357
3	·97582	·97701	70,779	74,773	72,745	71,882	3,412,258
4	·98304	·98273	69,067	73,053	71.030	70,420	3,340,376
5	·98707	·98635	67,896	71,792	69,814	69,348	3,269,956
6	·98897	·98845	67,018	70,812	68,886	68,496	3,200,608
7	·99059	·99024	66,279	69,994	68,108	67,781	3,132,112
8	·99195	·99175	65,655	69,311	67,455	67,180	3,064,331
9	·99307	·99298	65,127	68,739	66,905	66,671	2,997,151
10	.99397	·99397	64,675	68.256	66,439	66,238	2,930,480
11	·99466	·99475	64,285	67,845	66.038	65,862	2,864,242
12	·99517	·99532	63,942	67,489	65,688	65,532	2.798,380
13	·99552	·99572	63,633	67,173	65,376	65,232	2,732,848
14	·99571	·99595	63,348	66,885	65,090	64,954	2,667,616
15	·99576	·99604	63,076	66,614	64,818	64,685	2,602,662
16	·99569	·99601	62,809	66,351	64,552	64,419	2,537,977
17	·99551	·99586	62,538	66,086	64,285	64,147	2,473.558
18	·99524	·99563	62,258	65,813	64,007	63,862	2,409,411
19	·99489	·99532	61,961	65,525	63,716	63,559	2,345,549
20	·99447	·99495	61,644	65,219	63,404	63,236	2,281,990
21	·99399	·99453	61,303	64,889	63,069	62,888	2,218,754
22	·99347	·99408	60,935	64,535	62,707	62,512	2,155,866
23	·99290	·99359	60,537	64,153	62,317	62,107	2,093,354
24	·99231	·99308	60,107	63.741	61,897	61,670	2,031,247
25	·99168	·99255	59,645	63.300	61,444	61,202	1,969.577
26	·99103	·99201	59,149	62.829	60,960	60,702	1,908,375
27	·99037	·99147	58,618	62,327	60,444	60,169	1,847,673
28	·98970	·99093	58,054	61,796	59,897	59,605	1,787,504
29	·98900	·99039	57,456	61,235	59,317	59,010	1,727,899
30	·98829	·98986	56,824	60,646	58,706	58,385	1,668,889
31	·98757	·98934	56,159	60,032	58,065	57,730	1,610,504
32	·98683	.98883	55,461	59,392	57,396	57,046	1,552,774
33	·98607	·98832	54,730	58,728	56,698	56,335	1,495,728
34	·98529	·98782	53,967	58.042	55,974	55,597	1,439,393
35	·98449	·98732	53,173	57,335	55,223	54,833	1,383,796
36	·98367	·98682	52,349	56.608	54,446	54,044	1,328,963
37	·98282	·98631	51,494	55,862	53,644	53,230	1,274.919
38	·98194	·98579	50,609	55,097	52,819	52,392	1,221,689
39	·98105	·98524	49,695	54,314	51,969	51,531	1,169,297
40	·98013	·98466	48,753	53,512	51,096	50,646	1,117,766
41	·97919	·98404	47,785	52,691	50,200	49,739	1,067,120
42	·97822	·98337	46,790	51,850	49,282	48,808	1,017,381
43	·97723	·98264	45.771	50,988	48,339	47,854	968,573
44	·97620	·98184	44,729	50,102	47,374	46,877	920,719
45	·97513	·98096	43,664	49,192	46,386	45,877	873,842
46	·97401	·97999	42,578	48,256	45,373	44,852	827,965
47	.97283	·97891	41,472	47.290	44,337	43,802	783,113
48	·97157	·97774	40,345	46,293	43,274	42,725	739,311
49	·97021	·97647	39,198	45.262	42,183	41,622	696,586
50	·96874	·97509	38,030	44,197	41,067	40,490	654,964
51	·96714	·97361	36,841	43,096	39,921	39.329	614,474
52	·96538	·97201	35,631	41,959	38,747	38,140	575,145
53	·96346	·97029	34.397	40,784	37.542	36,919	537,005
54	·96135	·96845	33.140	3[illegible].573	36,307	35,669	500,086

MANCHESTER LIFE TABLE.

Based on the Mortality in Ten Years, 1881–1890.

Table 3.—Males and Females.—*(Continued.)*

Age	Chance of Living one Year from each age p_x		Of 100,000 Males born, the Number surviving at each Age	Of 100,000 Females born, the Number surviving at each Age	Of 100,000 of both Sexes (50,764 Males + 49,236 Females) born		
x	Males	Females			The Number surviving at each Age l_x	Population, or Years of Life lived in each Year P_x	Population, or Years of Life lived in and above each Year Q_x
55	·95904	·96646	31,859	38,324	35,042	34,388	464,417
56	·95652	·96432	30,555	37,039	33,747	33,078	430,029
57	·95379	·96201	29,226	35,717	32,422	31,739	396,951
58	·95083	·95950	27,876	34,360	31,069	30,370	365,212
59	·94763	·95677	26,505	32,969	29,688	28,976	334,842
60	·94418	·95385	25,117	31,544	28,281	27,557	305,866
61	·94048	·95072	23,715	30,088	26,853	26,119	278,309
62	·93652	·94731	22,303	28,605	25,406	24,665	252,190
63	·93230	·94362	20,887	27,098	23,945	23,198	227,525
64	·92782	·93965	19,473	25,570	22,475	21,726	204,327
65	·92307	·93538	18,067	24,027	21,002	20,253	182,601
66	·91804	·93081	16,677	22,475	19,532	18,788	162,348
67	·91273	·92593	15,311	20,920	18,072	17,336	143,560
68	·90711	·92075	13,974	19,370	16,631	15,907	126,224
69	·90119	·91526	12,676	17,835	15,216	14,510	110,317
70	·89497	·90945	11,424	16,324	13,836	13,151	95,807
71	·88842	·90334	10,224	14,846	12,499	11,839	82,656
72	·88155	·89692	9,083	13,411	11,214	10,582	70,817
73	·87435	·89022	8,007	12,028	9,987	9,389	60,235
74	·86681	·88326	7,001	10,708	8,826	8,264	50,846
75	·85891	·87615	6,069	9,458	7,738	7,214	42,582
76	·85065	·86866	5,212	8,287	6,726	6,243	35,368
77	·84204	·86101	4,434	7,198	5,794	5,354	29,125
78	·83307	·85314	3,734	6,198	4,946	4,549	23,771
79	·82374	·84508	3,110	5,288	4,182	3,826	19,222
80	·81405	·83682	2,562	4,468	3,501	3,186	15,396
81	·80400	·82839	2,086	3,739	2,900	2,625	12,210
82	·79358	·81982	1,677	3,098	2,376	2,139	9,585
83	·78280	·81111	1,331	2,539	1,926	1,724	7,446
84	·77167	·80228	1,042	2,060	1,543	1,373	5,722
85	·76018	·79336	804	1,652	1,222	1,081	4,349
86	·74833	·78436	611	1,311	955	840	3,268
87	·73615	·77531	457	1,028	738	645	2,428
88	·72362	·76622	337	797	564	489	1,783
89	·71077	·75712	244	611	425	366	1,294
90	·69759	·74803	173	463	316	270	928
91	·68410	·73897	121	346	231	197	658
92	·67031	·72995	83	256	168	142	461
93	·65624	·72101	55	187	120	101	319
94	·64189	·71216	36	135	84	71	218
95	·62729	·70342	23	96	59	49	147
96	·61244	·69481	15	67	40	34	98
97	·59738	·68636	9	47	28	23	64
98	·58211	·67804	5	32	19	15	41
99	·56665	·67001	3	22	13	10	26
100	·55103	·66215	2	15	8	6	16
101	...	·65452	1	10	5	4	10
102	...	·64715	0	6	3	3	6
103	...	·64005	...	4	2	2	3
104	...	·63325	...	2	1	1	1
105	...	...	...	1	0	0	0